This is a rough outline of a course to be taught at Hebrew University in May-June, 2023.

Prerequisites: A first course in algebraic geometry, along the lines of Hartshorne (Ch. 2-3) or Griffiths and Harris. Some familiarity with $p$-adic fields will be helpful.

The goal of the course is to introduce the various cohomology theories in algebraic geometry: de Rham cohomology and Hodge theory; étale cohomology; crystalline cohomology and $p$-adic Hodge theory. We will introduce all of these in the context of algebraic curves, where the theory of abelian varieties makes everything more concrete.

(1) De Rham cohomology and Hodge theory.
   (a) Algebraic curves over $\mathbb{C}$; the Riemann–Hurwitz formula; holomorphic differentials.
   (b) Integration of holomorphic differentials and the Riemann bilinear relations.
   (c) The Abel–Jacobi theorem.
   (d) Complex tori, theta functions, and the Riemann bilinear relations (again).
   (e) The Jacobian as an algebraic variety.

(2) Étale cohomology and Galois representations.
   (a) The Tate module of an abelian variety.
   (b) Formal properties of étale cohomology.
   (c) Galois representations.
   (d) Curves and abelian varieties over finite fields.
   (e) The Weil conjectures.
   (f) Global Galois representations again; Faltings’s finiteness lemma.

(3) Crystalline cohomology and $p$-adic Hodge theory.
   (a) De Rham cohomology over $p$-adic fields; functoriality in the special fiber; crystalline cohomology.
   (b) Finite flat group schemes.
   (c) Dieudonné modules.
   (d) The comparison isomorphism for abelian varieties over $\mathbb{Q}_p$. 