GEOMETRIC LAW FOR MULTIPLE RETURNS UNTIL A HAZARD

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ABSTRACT. For a ψ-mixing stationary process $\xi_0, \xi_1, \xi_2, ...$ we consider the number $\mathcal{N}_N$ of multiple recurrencies $\{\xi_{q_i(n)} \in \Gamma_N,\, i = 1, ..., \ell\}$ to a set Γ_N for n until the moment τ_N (which we call a hazard) when another multiple recurrence $\{\xi_{q_i(n)} \in \Delta_N,\, i = 1, ..., \ell\}$ takes place for the first time where $\Gamma_N \cap \Delta_N = \emptyset$ and $q_i(n) < q_{i+1}(n)$, $i = 1, ..., \ell$ are nonnegative increasing functions taking on integer values on integers. It turns out that if $P\{\xi_0 \in \Gamma_N\}$ and $P\{\xi_0 \in \Delta_N\}$ decay in N with the same speed then $\mathcal{N}_N$ converges weakly to a geometrically distributed random variable. We obtain also a similar result in the dynamical systems setup considering a ψ-mixing shift T on a sequence space Ω and study the number of multiple recurrencies $\{T^{q_i(n)}\omega \in A^a_m,\, i = 1, ..., \ell\}$ until the first occurence of another multiple recurrence $\{T^{q_i(n)}\omega \in A^b_m,\, i = 1, ..., \ell\}$ where A^a_m, A^b_m are cylinder sets of length m constructed by sequences $a, b \in \Omega$, respectively. We consider also ϕ-mixing shifts which are important since they allow to include some Young towers and Markov shifts with countable state space. In this case we obtain geometric limit law for the number of single returns to a cylinder until the first return to another cylinder and the case of multiple returns under ϕ-mixing remains open. This work is motivated by a number of papers on asymptotics of numbers of single and multiple returns to shrinking sets, as well as by the papers on open systems studying their behavior until an exit through a "hole".

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Date: August 23, 2018.

2000 *Mathematics Subject Classification.* Primary: 60F05 Secondary: 37D35, 60J05.

Key words and phrases. Geometric distribution, Poisson distribution, multiple returns, non-conventional sums, ψ-mixing, stationary process, shifts.