Fall 2022 graduate course 67648

Iterative and Randomized Matrix Algorithms

Lecturer: Prof. Omri Weinstein
Time: Wednesday 15:00 – 18:00

Modern data analysis and algorithm design—from convex optimization and network routing, to deep learning training and inference—rely on iterative *matrix operations* in high dimensions, such as matrix-multiplication and inverse, solving liner systems, singular-value decomposition, preconditioning and projections. Accelerating these expensive operations is crucial for speeding-up iterative optimization methods, and for large-scale inference and search in massive datasets.

This advanced course will explore the **interplay between iterative optimization methods and randomized linear algebra** ("sketching") as a tool for speeding-up continuous optimization methods, from gradient descent and interior-point (Newton) methods, to projection-free algorithms. We will apply these tools to a variety of important problems, from Linear Regression to Linear and Semidefinite programming. We will also explore the increasing role of dynamic data structures in speeding-up iterative methods.

This course can be viewed as Advanced Algorithms course, with a focus on matrix and optimization algorithms. It is a graduate-level course, geared toward Math and Theoretical CS students, with solid foundations in linear algebra and probability theory. No background in optimization will be assumed.

Grading: Students will be required to cover a topic of the course in depth, and present a short seminar on the subject (100% of the grade). Occasional Homework exercises will be given, but will not be graded. Minimum attendance policy: attendance in >70% lectures is required.

Tentative Topics:

- Linear Sketching (JL dimension reduction, row sampling, sparse recovery, leverage scores)
- Fast Approximate Matrix-Multiplication, Online Matrix Multiplication.
- Least-squares regression: Gradient Descent, Preconditioning, Conjugate-Gradient Descent, Chebychev iteration and Jacobi iteration (SDD). Tree Linear systems and Laplacians.
- Interior-Point Methods for Linear and Semidefinite Programming: Barrier methods, convergence analysis and Lewis weights (John's Ellipsoid). [Boyd book]
- Introduction to Conic programming: LPs and SDPs (Motivating examples: Flow LPs, Lovasz Thetha function, MaxCut and the CutNorm SDP)

- Projection-Free methods: The (Matrix) Mutiplicative-Weight Updates framework, projection-free algorithms (Frank-Wolfe & Trace-bounded optimization). Application to Multicommodity Flow.
- Interior-Point Methods: Newton steps, affine-invariance, Dikin Ellipsoid, Barrier Functions and the Central Path.
- Lower Bounds: (Nesterov's classic LB for 1st order methods, LB on IPM iterations)

*(Time Permitting: Mirror Descent (Bregman Projections), Accelerating cost-per-iteration via Dynamic Data Structures, Max Volume Ellipsoids / Lewis Weights)

Reference Courses (taught at other universities) :

- Advanced Algorithms (Ankur Moitra, MIT) : http://people.csail.mit.edu/moitra/854.html
- Interplay between Convex Optimization and Geometry (Yintat Lee, UW) : https://yintat.com/teaching/cse599-winter18/